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Speed limits to information erasure considering synchronization between heat transport and work cost

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A B S T R A C T

With the rapid development of the information technology (IT), heat transport and work cost in the information processing system gain significant scientific interest, but their synchronization problem has not been investigated hitherto. By the virtue of the finite-time Landauer principle, we here study the scenarios wherein one-dimensional Fourier heat transport synchronizes with the work cost of non-quasistatic information erasure. It is demonstrated that in such scenarios, the steady-state temperature distribution and harmonic temperature wave will respectively impose certain speed limits to information erasure, which give upper bounds on the amount of bits erased per unit time. The underlying physics of these speed limits are respectively the local well-definedness of the absolute temperature and the unsteadiness of the harmonic temperature wave. In engineering, the speed limit imposed by the steady-state temperature distribution can be understood as a performance limitation concomitant with the temperature stabilization, which quantitatively reveals the cost of the temperature stabilization.

1. Introduction

Over the past decades, the self-heating problem of the information processing system is increasingly severe and has already hindered the continued development of the information technology (IT) [1–3]. Analyses and resolutions of this problem therefore attract intensive attention in different research fields. On the one hand, in the field of heat transport, great efforts are denoted to modeling heat transport from the nanoscale to the macroscale [4–6], seeking the thermal interface material with high thermal conductivity and low elastic modulus [7–9], optimizing the heat transfer processes [10–15], and so on.

On the other hand, in the field of information thermodynamics, self-heating in the information processing system is attributed to the work cost of information erasure (bit reset) [16–24], and the present research focus is to estimate such work cost. The most fundamental estimation is the Landauer principle [17], which states that the minimal work cost of erasing one bit of information equals $k_B T\ln 2$. Here, k_B is the Boltzmann constant, and T denotes the absolute temperature of the position wherein information erasure takes place. This minimal work cost is usually termed as the Landauer bound or Landauer limit. The Landauer bound can be attained only for quasistatic information erasure, but the IT nowadays calls for information erasure at high speeds. Such inconsistency motivates researchers to explore the finite-time effect on the work cost of information erasure, and the so-called finite-time Landauer principle [25–28] is subsequently presented, namely,

$$W = (k_B\ln 2 + \sigma)T \geq (k_B\ln 2 + \sigma_0)T, \tag{1}$$

where W is the work cost of erasing one bit of information, σ is the averaged entropy generation due to the finite-time protocol, and σ_0 is a positive constant determined by the protocol duration and the dynamic parameters of the system. In general, σ_0 depends on the accuracy of information erasure [29,30] and decays with the increasing protocol duration [28,31]. For the small protocol duration, the decay is typically dominated by the exponential scaling, while for the large protocol duration, the inversely linear scaling dominates [31]. Moreover, a recent experimental study has demonstrated that the work distributions obey the trajectory-class fluctuation theorems [32].

The work cost of information erasure poses self-heating, and heat transport arises from self-heating. This means that heat transport in the information processing system is essentially induced by the work cost of information erasure. Meanwhile, the finite-time Landauer principle illustrates that the temperature plays an important role in the work cost of information erasure, whose spatial distribution and temporal evolution will be strongly influenced by the heat transport process. Thus, heat

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transport will in turn influence the work cost of information erasure. These facts imply that there can be a certain synchronization between heat transport and the work cost in the information processing system. However, such synchronization have not been discussed hitherto, and we here in the information processing system which satisfies the following assumptions. The first assumption is that in the system, the work cost of non-quasistatic information erasure coexists with one-dimensional heat conduction posed on the space domain $[0, L]$. The physical meaning of such space domain is that the information processing system is considered as a sheet, and heat conduction takes place only in the direction of its thickness. Second, the absolute temperature can be locally defined in the sense of local equilibrium, and based on such locally defined temperature, heat conduction can be modeled in terms of classical Fourier's law

$$q = -\kappa\frac{\partial T}{\partial x}, \tag{2}$$

where $q = q(x, t)$ is the heat flux, and κ is the thermal conductivity. As the most commonly used model of heat conduction, Fourier's law is generally valid for the macroscale heat conduction process [6]. More precisely, the validity of Fourier's law requires that the characteristic size and time of the heat conduction process are much larger than the mean free path and relaxation time of the heat carriers respectively. This requirement is a microscopic description of the second assumption. Third, the work cost per bit erased attains the lower bound in the finite-time Landauer principle, and mathematically, this assumption can be written as

$$\Phi = (k_B\ln 2 + \sigma_0)v_{bit}T. \tag{3}$$

Here, $\Phi = \Phi(x, t)$ is the power consumption per unit volume, $v_{bit} = v_{bit}(x, t)$ is the amount of bits that is erased per unit time per unit volume, which can be viewed as the local rate of information erasure. The fourth assumption is that the time needed by converting work into heat is negligibly small. This assumption allows us to equate the power consumption per unit volume and the intensity of the internal heat source, leading to

$$c\frac{\partial T}{\partial t} = -\frac{\partial q}{\partial x} + (k_B\ln 2 + \sigma_0)v_{bit}T, \tag{4}$$

with c the specific heat per unit volume. Through combining Eqs. (2) and (4), we arrive at the governing equation with respect to the locally defined absolute temperature, namely,

$$\frac{\partial T}{\partial t} = D\nabla^2 T + \frac{(k_B\ln 2 + \sigma_0)v_{bit}}{c}T, \tag{5}$$

with $D = \kappa/c$ the thermal diffusivity. Finally, v_{bit} is assumed as a positive constant for the sake of simplification, so the intensity of the internal heat source is proportional to the locally defined absolute temperature.

Based on the aforementioned assumptions, we first concern the synchronization problem between the work cost of information erasure and steady-state heat transport, and demonstrate that for such synchronization, the temperature distribution will impose a speed limit to information erasure. Moreover, the speed limit is discussed from the viewpoints of physics and engineering. Then, we investigate the synchronization problem between the work cost of information erasure and wave-like heat transport, including the spatially attenuated and temporally periodic (SATP) and temporally attenuated and spatially periodic (TASP) forms [33–35]. It is shown that such synchronization must be paired with another speed limit to information erasure, whose underlying physics is compared with that of the previous speed limit.

2. Speed limit imposed by steady-state temperature distribution

2.1. Mathematical derivation

We first consider the case wherein the work cost of information erasure synchronizes with steady-state heat transport, and in this case, Eq. (5) reduces to

$$\kappa\nabla^2 T + (k_B\ln 2 + \sigma_0)v_{bit}T = 0, \tag{6}$$

whose general solution is

$$T = C_1\sin\left[x\sqrt{\frac{(\sigma_0 + k_B\ln 2)v_{bit}}{\kappa}}\right] + C_2\cos\left[x\sqrt{\frac{(\sigma_0 + k_B\ln 2)v_{bit}}{\kappa}}\right]. \tag{7}$$

In Eq. (7), the coefficients C_1 and C_2 are determined by the boundary condition, and we here concern two types of the boundary conditions. The first type is the Dirichlet boundary condition

$$T(x=0) = T_0, \quad T(x=L) = T_L, \tag{8a}$$

where T_0 and T_L are constants. According to the third law of thermodynamics, T_0 and T_L must be strictly positive. The second type is the Neumann boundary condition

$$\left(\kappa\frac{\partial T}{\partial x}\right)\Bigg|_{x=0} = q_0, \quad \left(-\kappa\frac{\partial T}{\partial x}\right)\Bigg|_{x=L} = q_L, \tag{8b}$$

where q_0 and q_L are constants. In practical engineering, the boundary heat flux is generally for the purpose of cooling rather than heating the system. In the light of this, q_0 and q_L are restricted to non-negative numbers.

Substituting Eq. (8a) into Eq. (6) yields

$$\begin{cases} C_2 = T_0 \\ C_1\sin\alpha + C_2\cos\alpha = T_L \end{cases}, \tag{9}$$

with $\alpha = \sqrt{\frac{(\sigma_0 + k_B\ln 2)v_{bit}L^2}{\kappa}}$. For an arbitrary positive integer n, C_1 is mathematically arbitrary as long as the local rate of information erasure satisfies

$$v_{bit} = \frac{n^2\pi^2\kappa}{(k_B\ln 2 + \sigma_0)L^2}. \tag{10a}$$

Meanwhile, C_2 does not exist unless $T_0\cos\alpha = T_L$. Such C_1 and C_2 will give rise to a non-unique or non-existent temperature distribution, which is physically meaningless. Thus, the local rate of information erasure must be constrained as

$$v_{bit} \neq \frac{n^2\pi^2\kappa}{(k_B\ln 2 + \sigma_0)L^2}, \quad \forall n \in \mathbb{N}^+. \tag{10b}$$

Inequality (10b) can be regarded as the existence and uniqueness condition of the steady-state temperature distribution.

With inequality (10b) satisfied, the solution of Eq. (9) is

$$C_1 = T_L\csc\alpha - T_0\cot\alpha, \quad C_2 = T_0, \tag{11}$$

and the corresponding temperature distribution reads

$$\begin{aligned} T &= T_L\csc\alpha\sin\left(\frac{\alpha x}{L}\right) + T_0\left[\cos\left(\frac{\alpha x}{L}\right) - \cot\alpha\sin\left(\frac{\alpha x}{L}\right)\right] \\ &= \sqrt{(T_L^2 + T_0^2)\csc^2\alpha - 2T_0T_L\csc\alpha\cot\alpha} \\ &\quad \sin\left[\frac{\alpha x}{L} + \mathrm{arccot}\left(\frac{T_L\csc\alpha}{T_0} - \cot\alpha\right)\right]. \end{aligned} \tag{12}$$

Besides the uniqueness and existence, a physically meaningful temperature distribution must obey the third law of thermodynamics as well. The mathematical statement of obeying the third law of thermodynamics is written as

$$T = T(x) > 0, \quad \forall x \in [0, L]. \tag{13}$$

For the temperature distribution given by Eq. (12), inequality (13) is equivalent to

$$f_1(x) = \sin\left[\frac{\alpha x}{L} + \operatorname{arccot}\left(\frac{T_L \csc\alpha}{T_0} - \cot\alpha\right)\right] > 0, \quad \forall x \in [0, L]. \tag{14}$$

As a sinusoidal function, $f_1(x)$ will have at least one zero point when the angular variation on $[0, L]$ is equal to or larger than π. As a consequence, inequality (14) has the following necessary condition

$$\alpha < \alpha_{i=1} = \pi. \tag{15}$$

Indeed, inequality (15) is not only the necessary condition but also the sufficient condition for inequality (14), whose proof is as follows. In the presence of inequality (15), $f_1(x)$ is monotonic if the boundary temperatures satisfy

$$\max\{T_L\cos\alpha, T_0\cos\alpha\} \geq \min\{T_L, T_0\}. \tag{16}$$

Owing to the monotonicity, the minimum of $f_1(x)$ appears at the boundary, so the sufficient condition for inequality (14) is composed of

$$f_1(x=0) = \frac{T_0}{\sqrt{(T_L^2 + T_0^2)\csc^2\alpha - 2T_0T_L\csc\alpha\cot\alpha}} > 0, \tag{17a}$$

and

$$f_1(x=L) = \frac{T_L}{\sqrt{(T_L^2 + T_0^2)\csc^2\alpha - 2T_0T_L\csc\alpha\cot\alpha}} > 0. \tag{17b}$$

Because both of T_0 and T_L are strictly positive, inequalities 17a) and (17b) hold automatically. Accordingly, inequality ((15) is the sufficient condition for inequality (14) when the boundary temperature satisfy inequality (16). The other case is

$$\max\{T_L\cos\alpha, T_0\cos\alpha\}\langle\min\{T_L, T_0\}, \tag{18}$$

and in this case, $f_1(x)$ first increases and then decreases. Consequently, the minimum of $f_1(x)$ still equals $f_1(x=0)$ or $f_1(x=L)$, and inequality (15) remains the sufficient condition for inequality (14). Inequality (15) has now proved to be the necessary and sufficient condition for inequality (14). This means that under Dirichlet boundary condition, the steady-state temperature distribution obeys the third law of thermodynamics if and only if inequality (15) is satisfied.

To sum up, the steady-state temperature distribution will not be physically meaningful unless both of inequalities 10b) and ((15) are satisfied. Mathematically, this will enforce the intersection of inequalities 10b) and ((15), which is calculated as:

$$v_{bit} < v_{i=1} = \frac{\pi^2\kappa}{(\sigma_0 + k_B\ln 2)L^2}. \tag{19}$$

Furthermore, the spatial integration of v_{bit} actually equals the amount of bits erased per unit time, namely,

$$\frac{dI_{bit}}{dt} = v_{bit}AL, \tag{20}$$

where I_{bit} is the amount of bits erased in the system, and A is the cross-sectional area. Then, inequality (19) can be reformulated as

$$\frac{dI_{bit}}{dt} < V_{i=1} = \frac{\pi^2\kappa A}{(k_B\ln 2 + \sigma_0)L}. \tag{21}$$

$\frac{dI_{bit}}{dt}$ actually characterizes the speed of information erasure, so inequality (21) is a speed limit to information erasure.

Under the Neumann boundary condition, C_1 and C_2 are governed by

$$\begin{cases} \kappa\alpha C_1 = q_0 L \\ \kappa\alpha(C_2\sin\alpha - C_1\cos\alpha) = q_L L \end{cases}. \tag{22}$$

For this system of equations, arbitrary C_2 and non-existent C_1 will occur when the local rate of information erasure satisfies inequality (10a). Hence, inequality (10b) remains the uniqueness and existence condition of the steady-state temperature distribution.

When inequality (10b) is satisfied, we can acquire

$$C_1 = \frac{q_0 L}{\kappa\alpha}, \quad C_2 = \frac{(q_0\cot\alpha + q_L\csc\alpha)L}{\kappa\alpha} \tag{23}$$

and

$$\begin{aligned} T &= \frac{L}{\kappa\alpha}\left\{q_L\csc\alpha\cos\left(\frac{\alpha x}{L}\right) + q_0\left[\cos\left(\frac{\alpha x}{L}\right)\cot\alpha + \sin\left(\frac{\alpha x}{L}\right)\right]\right\} \\ &= \frac{L\sqrt{(q_L^2 + q_0^2)\csc^2\alpha + 2q_0q_L\csc\alpha\cot\alpha}}{\kappa\alpha}\sin\left[\frac{\alpha x}{L} + \operatorname{arccot}\left(\frac{q_0\sin\alpha}{q_L + q_0\cos\alpha}\right)\right]. \end{aligned} \tag{24}$$

Likewise, the steady-state temperature distribution corresponding to the Neumann boundary condition must obey the third law of thermodynamics stated by inequality (13). For the temperature distribution given by Eq. (24), the necessary and sufficient condition for inequality (13) is composed of

$$(q_L^2 + q_0^2)\csc^2\alpha + 2q_0q_L\csc\alpha\cot\alpha > 0 \Leftrightarrow q_L^2 + q_0^2 > 0, \tag{25}$$

and

$$f_2(x) = \sin\left[\frac{\alpha x}{L} + \operatorname{arccot}\left(\frac{q_0\sin\alpha}{q_L + q_0\cos\alpha}\right)\right] > 0, \quad \forall x \in [0, L]. \tag{26}$$

Indeed, inequality (25) indicates that the temperature distribution cannot be steady-state when both surfaces of the system are adiabatic. This is physically reasonable because in the absence of boundary cooling, the balance of the internal energy can never be maintained. Just like $f_1(x)$, $f_2(x)$ is a sinusoidal function whose angular variation on $[0, L]$ equals π, so inequality (15) is also the necessary condition for inequality (26). When it comes to the sufficient condition for inequality (26), there two cases. If neither surface of the system is adiabatic, $f_2(x)$ first increases and then decreases. As a result, the sufficient condition for inequality (26) is given by

$$\begin{cases} f_2(x=0) = \dfrac{q_L\csc\alpha + q_0\cot\alpha}{\sqrt{(q_L^2 + q_0^2)\csc^2\alpha + 2q_0q_L\csc\alpha\cot\alpha}} > 0 \\ f_2(x=L) = \dfrac{q_0\csc\alpha + q_L\cot\alpha}{\sqrt{(q_L^2 + q_0^2)\csc^2\alpha + 2q_0q_L\csc\alpha\cot\alpha}} > 0 \end{cases}, \tag{27}$$

By combining inequality (15) and condition (27), the necessary and sufficient condition for inequality (26) can be obtained as

$$\alpha < \alpha_{i=2} = \arccos\left(-\min\left\{\frac{q_0}{q_L}, \frac{q_L}{q_0}\right\}\right). \tag{28}$$

If the system has only one adiabatic surface, $f_2(x)$ is monotonic and its minimum will appear at the boundary. In this case, the sufficient condition for inequality (26) is still inequality (27), but the combination (15) and (27) will lead to a different result from inequality (28), namely,

$$\alpha < \alpha_{i=3} = \frac{\pi}{2}. \tag{29}$$

We recall that inequality (25) excludes the case wherein both surfaces are adiabatic. Based on inequalities (25), 28) and ((29), the necessary and sufficient condition for inequality (13) can be summarized as follows,

$$\begin{cases} \alpha < \alpha_{i=k+2} \\ k < 2 \end{cases}, \tag{30}$$

where k is the number of adiabatic surfaces. Under the Neumann boundary condition, the steady-state temperature distribution obeys the third law of thermodynamics if and only if condition (30) is satisfied.

Similarly, the steady-state temperature distribution is physically meaningful only when the intersection of inequalities 10b) and ((28) is satisfied, namely,

$$\begin{cases} v_{bit} < v_{i=k+2} = \dfrac{\kappa\alpha^2_{i=k+2}}{(k_B\ln 2+\sigma_0)L^2}. \\ k < 2 \end{cases} \tag{31}$$

As Eq. (20) remains unchanged, $v_{bit} < v_{i=k+2}$ is equivalent to

$$\frac{dI_{bit}}{dt} < V_{i=k+2} = \begin{cases} \dfrac{\pi^2\kappa A}{4(k_B\ln 2+\sigma_0)L}, & k=0 \\ \dfrac{\kappa A}{(k_B\ln 2+\sigma_0)L}\left[\arccos\left(-\min\left\{\dfrac{q_0}{q_L},\dfrac{q_L}{q_0}\right\}\right)\right]^2, & k=1 \end{cases}, \tag{32}$$

which also gives an upper bound on the speed of information erasure.

2.2. Discussion from viewpoints of physics and engineering

Although the mathematical formulations of the speed limit will vary with the boundary condition, each formulation gives an upper bound on the speed of information erasure. The physical meaning of such speed limit is that the temperature distribution can be steady-state only when information erasure is sufficiently slow. Furthermore, the speed limit is derived from three mathematical properties of the temperature distribution, including the existence, uniqueness and positivity (the third law of thermodynamics). All of these mathematical properties arise from a physical assumption that the absolute temperature is locally well-defined. Consequently, the underlying physics of the speed limit is the local well-definedness of the absolute temperature. The upper bounds in these speed limits depend on only not the boundary conditions but also the features of the system, i.e., σ_0 and κ. That is because to maintain a steady-state temperature distribution, all heat generated by information erasure must be transferred to the environment. To this end, all heat generated by information erasure must first be able to arrive at the boundary of the system, and such ability relies on the features of the system including σ_0 and κ. Then, all heat must be able to flow from the boundary to the environment, which relies on the boundary condition. Moreover, the upper bound V_i can be factorized as

$$\begin{aligned} V_i &= \left(\frac{A}{L}\right)\times(B_i\kappa)\times\left(\frac{1}{k_B\ln 2+\sigma_0}\right), \\ B_i &= \begin{cases} \pi^2, & i=1 \\ \left[\arccos\left(-\min\left\{\dfrac{q_0}{q_L},\dfrac{q_L}{q_0}\right\}\right)\right]^2, & i=2\,. \\ \pi^2/4, & i=3 \end{cases} \end{aligned} \tag{33}$$

The first factor is nothing but a geometrical parameter of the system, which is always finite. The second factor is proportional to the thermal conductivity κ, a thermophysical property measuring the ability to conduct heat, and the proportionality coefficient B_i depends on the boundary condition of the heat conduction process. This factor can never reach infinity unless the thermal conductivity is infinitely large, which actually enforces thermodynamically reversible heat conduction. Specifically, for a given $V_{i=2}$, $B_{i=2}$ can also be viewed as a restriction on the Neumann boundary condition. The third factor can diverge to infinity as long as $(k_B\ln 2+\sigma_0)$ vanishes. Nevertheless, $(k_B\ln 2+\sigma_0)$ is the entropy generation by erasing one bit of information, so it must be strictly positive due to the thermodynamic irreversibility of information erasure. Taking all these into account, the finiteness of V_i has a thermodynamic meaning, namely that both of the heat conduction and information erasure processes are thermodynamically irreversible.

We now discuss the speed limit from the viewpoint of engineering. In engineering, stabilizing the temperature distribution is a common demand because the performance and reliability of the practical device are strongly influenced by the operating temperature. As demonstrated above, such temperature stabilization will inevitably impose the speed limit to information erasure, which can be understood as a concomitant performance limitation. The speed limit understood as the performance limitation quantitatively reveals the cost of the temperature stabilization, and implies that the temperature stabilization can be optimized via increasing V_i. Such a optimization allows us to reduce the cost of the temperature stabilization, and can be realized by increasing κ, σ_0^{-1} and B_i. Compared with increasing κ and σ_0^{-1}, increasing B_i is more feasible because the choice of the boundary condition is independent of the system parameters. Mathematically, it can be proved that B_1, B_2 and B_3 fulfill the following inequality chain

$$B_1 = B_2(q_0 = q_L)\rangle B_2(q_0 \neq q_L)\rangle B_3. \tag{34}$$

According to this inequality chain, the Dirichlet boundary condition and the Neumann boundary condition with $q_0 = q_L$ are the first-choice boundary conditions. This inequality chain also indicates that for the Neumann boundary condition, no adiabatic surface is always superior to one adiabatic surface.

3. Speed limit imposed by harmonic temperature wave

Indeed, the speed limit to information erasure can also emerge from the synchronization between the work cost and unsteady-state heat transport. In order to illustrate this, we consider the following harmonic temperature wave [33–35],

$$T_j(x,t) = \phi(x) + \theta_j(x,t), \quad \theta_j(x,t) = \begin{cases} T^*\exp(\mathbf{i}kx - \mu x - \mathbf{i}\omega t), & j=1 \\ T^*\exp(\mathbf{i}kx - \mathbf{i}\omega t - \xi t), & j=2 \end{cases}, \tag{35}$$

where $\phi(x)$, $\theta_{j=1}(x,t)$, $\theta_{j=2}(x,t)$, T^*, k, ω, μ, and ξ are, respectively, a positive solution of Eq. (6), the SATP form, the TASP form, the complex amplitude, the wavenumber, the angular frequency, the spatial attenuation exponent, and the temporal attenuation exponent. In order to guarantee the validity of Fourier's law, ω and ξ must be sufficiently small. Otherwise, the characteristic time of the heat conduction process may be comparable to or even smaller than the relaxation time of the heat carriers. Of course, this requirement may not hold in practical problems. As a result, the following discussion should be restricted to the temperature control problem wherein ω and ξ are enforced to be sufficiently small. It should be noted that the harmonic temperature wave will vanish for steady-state $\theta_j(x,t)$. Therefore, the harmonic temperature wave with the SATP form exists only if $\omega > 0$, and for the harmonic temperature wave with the TASP from, its existence needs $\omega^2 + \xi^2 > 0$.

Substituting $T_j(x,t)$ into Eq. (5) yields

$$0 = \begin{cases} \mu^2 - k^2 + \dfrac{\alpha^2}{L^2} + \mathbf{i}\left(\dfrac{\omega}{D} - 2k\mu\right), & j=1 \\ \dfrac{\alpha^2}{L^2} + \dfrac{\xi}{D} - k^2 + \dfrac{\mathbf{i}\omega}{D}, & j=2 \end{cases}, \tag{36}$$

which enforces

$$0 = \begin{cases} \mu^2 - k^2 + \dfrac{\alpha^2}{L^2} = \dfrac{\omega}{D} - 2k\mu, & j=1 \\ \dfrac{\alpha^2}{L^2} + \dfrac{\xi}{D} - k^2 = \dfrac{\omega}{D}, & j=2 \end{cases}. \tag{37}$$

By the virtue of Eq. (37), ω, μ and ξ can be expressed in terms of k, namely,

$$\omega(k, j=1) = \begin{cases} 2Dk\sqrt{k^2 - \alpha^2 L^{-2}}, & k > \alpha L^{-1} \\ 0, & 0 < k \le \alpha L^{-1} \end{cases}, \tag{38a}$$

$$\mu(k,j=1) = \begin{cases} \sqrt{k^2 - \alpha^2 L^{-2}}, & k > \alpha L^{-1} \\ 0, & 0 < k \le \alpha L^{-1} \end{cases}, \tag{38b}$$

$$\omega(k,j=2) = 0, \quad k > 0, \tag{38c}$$

$$\xi(k,j=2) = \begin{cases} D\left(k^2 - \alpha^2 L^{-2}\right), & k > \alpha L^{-1} \\ 0, & 0 < k \le \alpha L^{-1} \end{cases}, \tag{38d}$$

By combining Eq. (38a–d) with $\omega > 0$ and $\omega^2 + \xi^2 > 0$, we can deduce $k > \alpha L^{-1}$, which is equivalent to the following speed limit,

$$\frac{dI_{bit}}{dt} < V_j = \frac{\kappa A L k^2}{k_B \ln 2 + \sigma_0}, \tag{39}$$

which also gives an upper bound on the speed of information erasure. For given σ_0 and κ, V_j is identically larger than V_i as long as the wavenumber satisfies $k > \pi L^{-1}$. This implies that the temperature wave with a sufficiently large wavenumber always allows a higher performance than the steady-state temperature distribution. Therefore, replacing the temperature stabilization by maintaining such temperature wave may serve as a strategy in thermal management.

Inequality (39) is actually entailed by unsteady-state $\theta_j(x,t)$, and its derivation does not involve the existence, uniqueness or positivity of the solution. Consequently, the underlying physics of this speed limit is the unsteadiness of the harmonic temperature wave rather than the local well-definedness of the absolute temperature, which differs from the speed limit imposed by the steady-state temperature distribution. Of course, $T_j(x,t)$ must possess the uniqueness and existence, and obey the third law of thermodynamics likewise. The uniqueness and existence are guaranteed by Eqs. (38a–d). When it comes to the third law of thermodynamics, its mathematical statement becomes

$$T_j(x,t) > 0, \quad \forall (x,t) \in [0,L] \times [0,+\infty), \tag{40}$$

which requires

$$\min_{(x,t)\in[0,L]\times[0,+\infty)}\left\{T_j(x,t)\right\} = \min_{x\in[0,L]}\{\phi(x)\} - |T^*| > 0 \Leftrightarrow |T^*| < \min_{x\in[0,L]}\{\phi(x)\}. \tag{41}$$

As a consequence, the third law of thermodynamics will entail an upper bound on the complex amplitude rather than the speed of information erasure.

As shown in Ref. [33], the SATP and TASP forms of the hyperbolic non-Fourier temperature wave exhibit the anomalous dispersions, which are characterized by $v_g > v_p$. Here, v_g and v_p are respectively the group velocity and the phase velocity of the temperature wave. As a comparison, we here also discuss whether the temperature waves in this work exhibit the anomalous dispersions. For the SATP form, the group velocity and the phase velocity are respectively given by

$$v_g = \frac{\partial\omega(k,j=1)}{\partial k} = \frac{2D\left(2k^2 - \alpha^2 L^{-2}\right)}{\sqrt{k^2 - \alpha^2 L^{-2}}}, \quad k > \alpha L^{-1}, \tag{42a}$$

$$v_g = \frac{\partial\omega(k,j=1)}{\partial k} = \frac{2D\left(2k^2 - \alpha^2 L^{-2}\right)}{\sqrt{k^2 - \alpha^2 L^{-2}}}, \quad k > \alpha L^{-1}, \tag{42b}$$

which satisfy

$$v_g = v_p\left(1 + \frac{k^2}{k^2 - \alpha^2 L^{-2}}\right) \rangle v_p. \tag{42c}$$

Therefore, the SATP form also exhibits the anomalous dispersion, which is a similarity between the temperature wave discussed in this work and the hyperbolic non-Fourier temperature wave. For the TASP form, we have $v_g = v_p = 0$, so the TASP form does not exhibit the anomalous dispersion. This is a difference between the temperature wave discussed in this work and the hyperbolic non-Fourier temperature wave.

4. Conclusions

- When one-dimensional Fourier heat conduction synchronizes with the work cost of non-quasistatic information erasure, the steady-state temperature distribution will inevitably impose a speed limit to information erasure, which gives an upper bound on the amount of bits erased per unit time. and dependent of the boundary condition.
- If the aforementioned speed limit is not satisfied, the steady-state temperature distribution will be physically meaningless due to the non-uniqueness, non-existence, or violation of the third law of thermodynamics. This means that the underlying physics of this speed limit is the local well-definedness of the absolute temperature. From the viewpoint of engineering, this speed limit can be understood as a performance limitation concomitant with the temperature stabilization, which quantitatively reveals the cost of the temperature stabilization.
- When one-dimensional Fourier heat conduction synchronizes with the work cost of non-quasistatic information erasure, the harmonic temperature wave, including the SATP and TASP forms, will impose another speed limit to information erasure, which also gives an upper bound on the amount of bits erased per unit time. The underlying physics of this speed limit is the unsteadiness of the harmonic temperature wave rather than the local well-definedness of the absolute temperature.

CRediT authorship contribution statement

Shu-Nan Li: Formal analysis, Writing – original draft. **Bing-Yang Cao:** Conceptualization, Writing – review & editing.

Declaration of Competing Interest

The authors declare no conflict of interest.

Data availability

No data was used for the research described in the article.

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Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ijheatmasstransfer.2023.124688.

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